Math 1B
Midterm 3 Review

For sections 5.4–5.5 and 7.1–7.5:

Get together a group of classmates.
Make a copy of the following pages and integrals:

5.4	5-46	7.1	3-45	7.4	7-54
5.5	7-73	7.2	1-35, 47-49, 51-52	7.5	1-82
5.REV	9-40	7.3	4-29	7.REV	1-38

Cut them up, so each question is on a separate slip of paper.
Throw them in a pile and mix them up.
One at a time, randomly pick out a question from the pile and everyone solve it independently.
Compare solutions and discuss which one is fastest/easiest, and how you can recognize what method to use.

The following questions act as a review for 7.8.

[1] Determine if the following integrals converge or diverge. If an integral converges, find its value.

[a] $\int_0^\infty x^2 e^{-3x}\,dx$ [b] $\int_0^\infty \frac{1}{\sqrt[3]{x-1}}\,dx$ [c] $\int_{-\infty}^\infty \frac{1}{x^2+4}\,dx$ [d] $\int_{-\infty}^\infty \frac{x}{x^2+4}\,dx$

[e] $\int_{-\infty}^0 \frac{e^x}{1+e^x}\,dx$ [f] $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ [g] $\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$ [h] $\int_0^1 \frac{1}{x(\ln x)^2}\,dx$

[i] $\int_0^\pi \tan x\,dx$

[2] Determine if the following integrals converge or diverge. Justify your answer.

[a] $\int_1^\infty \frac{2+\sin x}{x}\,dx$ [b] $\int_1^\infty \frac{2+\sin x}{x^2}\,dx$ [c] $\int_0^\infty e^{-x^2}\,dx$ [d] $\int_e^\infty \frac{1}{\ln x}\,dx$

[e] $\int_e^\infty \frac{1}{x\ln x}\,dx$ [f] $\int_2^\infty \frac{x+1}{\sqrt{x^4-1}}\,dx$ [g] $\int_1^\infty \frac{\cos^2 x}{xe^x}\,dx$

Answers

[1] [a] $\frac{2}{27}$ [b] diverges [c] $\frac{\pi}{2}$ [d] diverges

[e] $\ln 2$ [f] $\frac{\pi}{2}$ [g] 2 [h] diverges

[i] diverges

[2] [a] diverges – compare to $\frac{1}{x}$ [b] converges – compare to $\frac{3}{x^2}$

[c] converges – compare to e^{-x} [d] diverges – compare to $\frac{1}{x}$

[e] diverges – find antiderivative and take limit [f] diverges – compare to $\frac{1}{x}$

[g] converges – compare to $\frac{1}{e^x}$